Pressure Broadening of Multiply Ionized Carbon Lines

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In a helium plasma with carbon impurity, the pressure broadened profiles of seven C III and two C IV lines have been measured. The plasma which was produced by magnetic compression had typically an electron density of $4 \cdot 10^{17}$ cm^{-3}, a temperature of 60 000°K, and a life time of about 0.1 μs. Electron densities have been deduced from the width of the He II $\lambda = 3203$ Å line. If $\Delta E_{ii'} \ll k\,T$ ($\Delta E_{ii'}$ = energy difference between perturbed level i and perturbing level i') the classical straight line non-adiabatic theory gives values being in good agreement (error $< \pm 25\%$) with measured widths for six lines. For two lines the deviations are larger. For the C IV resonance line ($\lambda = 1550$ Å) with $\Delta E_{ii'} > k\,T$, the profile can be explained by hyperbolic path, adiabatic collision theory. The semiempirical Gaunt-factor approximation gives a rough estimate of the width, but normally the values obtained are too low.

A. Introduction

Numerous investigations have been carried out on the pressure broadening of neutral and singly ionized atom lines emitted from a plasma. Agreement between theory and experiment is with some exceptions within $\pm 20\%$ for the neutral [1], $\pm 50\%$ (or even more) for the singly ionized atoms [2, 3]. For multiply ionized ions, nearly no quantitative comparison between theory and experiment has been made although broadened lines of those ions have been observed in many laboratory and astrophysical plasmas. The main reason for this gap of information is probably that an accurate measurement of the electron density n_e was not possible in these plasmas because of strong density gradients. Therefore, a long plasma cylinder of about 30 cm length and 0.6 cm diameter was used here as a light source (see chapter: "Experimental Arrangement").

In this paper we describe measurements of the profiles of C III and C IV lines emitted from a magnetically compressed plasma whose electron density n_e is deduced from the width of the He II line $\lambda = 3203$ Å. Since C III (47 eV) and C IV (64 eV) have similar ionization energies as He II (54 eV), it can be expected that their lines are emitted from the same region of the plasma. Therefore the errors of the line width and n_e caused by inhomogeneous plasma layers should be low. The temperature T is normally not very critical for the interpretation of line broadening data. T has been estimated with sufficient accuracy from an absolute line intensity and from the appearance of a line intensity maximum as function of the voltage of the condenser bank.

B. Calculation of the Line Width

For the isolated ion lines observed here electron collisions are the main broadening mechanism [1–3] and a Lorentz intensity profile $I(\Delta\omega)$ or $I(\Delta\lambda)$ with a shifted maximum can be expected:

$$I(\Delta\omega) = \frac{w}{\pi} \frac{1}{(\Delta\omega - d)^2 + w^2}$$

or

$$I(\Delta\lambda) = \frac{\Delta\lambda_h}{2\pi} \frac{1}{(\Delta\lambda - \delta)^2 + (\Delta\lambda_h/2)^2}$$

where w = half-half width in angular frequency units, $\Delta\lambda_h$ = full halfwidth of the line in wavelength units, d and δ = line shift. Mainly two methods have been used for the calculation: the classical straight line and the semiempirical Gaunt factor approximation.

In the case $\Delta E_{ii'} \ll k\,T$ the hyperbolic orbit calculation can be replaced by the straight line approximation [2, 3]. The following equations for the broadening of a level i have been used [1]:

$$w_i + i\,d_i = \frac{4\pi}{3} n_e \left(\frac{\hbar}{m}\right)^2 \int \frac{dv}{v} f(v) \left\{ \tfrac{1}{2} \left| \sum_{i'} |\langle i|R|i'\rangle|^2 \cdot [A(z_{ii'}^{\min}) + i\,B(z_{ii'}^{\min})] \right| + \sum_{i'} |\langle i|R|i'\rangle|^2 [a(z_{ii'}^{\min}) + i\,b(z_{ii'}^{\min})] \right\}$$

and

$$z_{ii'}^{\min} = \frac{\omega_{ii'}\,\varrho_{\min}}{v} \approx \left(\frac{2}{3}\right)^{1/2} \frac{\Delta E_{ii'}}{m\,v^2} \left| \sum_{i,i'} |\langle i|R|i'\rangle|^2 [A(z_{ii'}^{\min}) + i\,B(z_{ii'}^{\min})] \right|^{1/2}$$

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with A, B, a, b functions defined in [1], $|\langle i|R|i'\rangle|^2$ = squares of matrix elements of the electron position vector taken from BATES and DAMGAARD [1,4], $\Delta E_{ii'}$ = energy difference between perturbed level i and perturbing level i'. Upper and lower level broadening has been taken into account. This means: $w = \Sigma\, w_i$ (i = lower or upper level). Only a limited number of perturbing levels i' can be included in the calculation. This causes an error smaller than 15% in the line width w.

The average over the velocity distribution has been obtained in an approximate manner taking $\int f(v)\, dv = 1$ and v a constant value, either

$$\text{a)}\quad v = (\overline{v^{-1}})^{-1} \quad \text{or} \quad \text{b)}\quad v = (\overline{v^2})^{1/2}.$$

The results had a discrepancy of more than 10% only in a few cases, when the calculations were anyhow less accurate, since the contributions of strong collisions were large.

For the C IV resonance line with $\Delta E_{ii'} > k\,T$ adiabatic processes are important and hyperbolic orbits have to be used. The calculations have been performed by SAHAL-BRÉCHOT and SEGRE [5]. Their value $\gamma = 2\,w = 0.4 \cdot 10^{-6}\, n_e\, s^{-1}$ for $T = 4 \cdot 10^4$ °K has been used.

GRIEM [2] gives the following semiempirical equation for the width W_{se} of an ion line

$$W_{se} \approx 8 \left(\frac{\pi}{3}\right)^{3/2} \frac{\hbar}{m\, a_0}\, n_e \left(\frac{E_H}{k\,T}\right)^{1/2} \cdot \left[\sum_{i'} |\langle i'|R|i\rangle|^2\, g_{se}\left(\frac{\frac{3}{2}\,k\,T}{|\Delta E_{i'i}|}\right) + \sum_{f'} |\langle f'|R|f\rangle|^2\, g_{se}\left(\frac{\frac{3}{2}\,k\,T}{|\Delta E_{f'f}|}\right)\right]$$

[E_H = ionization energy of hydrogen, $|\langle i'|R|i\rangle|^2$ and $|\langle f'|R|f\rangle|^2$ squares of matrix elements of the allowed transitions from the lower and upper level of the emitted line, $g_{se}(3\,k\,T/|\,2\,\Delta E_{i'i}|)$ = semiempirical Gaunt factor given graphically in [2]].

C. Evaluation of the Plasma Parameters

The electron density n_e has been determined by the halfwidth of the He II line $\lambda = 3203$ Å using the relation

$$n_e = C\,(\Delta\lambda_h)^{3/2}$$

with $C = 2.25 \cdot 10^{15}$ cm^{-3} Å$^{-3/2}$, which is a mean value between the experimental and theoretical [6] one. The Saha equation cannot be used for He II under the present conditions [6]. Using the method described in [6], it was calculated that due to multiple ionization of helium and carbon n_e is about 12% higher than n_i. The density was always $n_e = 4 \cdot 10^{17}$ cm^{-3}, except for the measurement of the $\lambda = 5801$ Å line, where $n_e = 6.9 \cdot 10^{17}$ cm^{-3} was used.

The degree of ionization of carbon was deduced as follows. The plasma temperature was varied by changing the capacitor voltage, and the intensities of the continuum, of the C III 4187 Å line and of the optically thin wings of the C IV resonance line were measured (Fig. 1). A maximum of α, the ratio of the number of C IV ions to the total number of C ions, as function of T is expected, which lies at somewhat higher T than the maximum of the C III line and at somewhat lower T than that of the C IV line. Figure 1 shows that this maximum should appear at a voltage of about 20.3 kV. This voltage was used since near its maximum the exact value of α is not very sensitive to the applied ionization formula. Assuming that the Saha equation can be applied we get $\alpha = 0.83$ and $T = 56\,000$ °K. The value of T has to be known only roughly for the line broadening calculations.

For the evaluation of the profile of the optically thick C IV resonance line, n_{i0} has to be known (n_{i0} = number of carbon ions in the C IV ground state cm^{-3}). This has been performed by two methods:

1. The carbon concentration in the initial gas is 2.8%. With $n_e = 1.12\, n_i$ (due to the double ionization, see above) the number density ratio of carbon ions to electrons is 2.5%. From this value and $n_e = 4 \cdot 10^{17}$ cm^{-3}, a carbon ion density of $1 \cdot 10^{16}$ cm^{-3} is deduced. With $\alpha = 0.83$ we obtain the C IV ion density $n_{C\,IV} = 8.3 \cdot 10^{15}$ cm^{-3}. Using Boltzmann's law for the distribution of C IV ions over the different C IV energy levels we find

$$n_{i0} = 0.625\, n_{C\,IV} = 5.2 \cdot 10^{15}\ \text{cm}^{-3}.$$

For this calculation T has to be known roughly.

2. Using the absolute intensity of C III $\lambda = 4187$ Å, a hydrogenlike oscillator strength, Saha's equation between C III 5 g^{1}G state and C IV ground

state, and $T = 56\,000$ °K, a value of $n_{i0} = 5.4 \cdot 10^{15}$ cm^{-3} was deduced. The agreement between the two derived values of n_{i0} indicates, that the Saha equation is sufficiently approximated for C III.

D. Experimental Arrangement

The experiment is similar to that described earlier by the author [6]. It is a small theta-pinch arrangement with a preionization, a preheating, and a main discharge using a 25 kV, 3 μF condenser bank. The compression coil has a length of 30 cm and a diameter of 4 cm. The filling pressure was about 0.4 Torr He + methane (0.2 – 6%).

Spectroscopic observations have been made axially at both ends. On one end, the plasma radiation is focussed in vacuum (i. e. filling pressure of the discharge tube) by a mirror on the slit of a 50 cm Ebert-vacuum monochromator, on the other end it is focussed in air by a quartz-fluorite achromat on the slit of another 50 cm Ebert monochromator equipped with a photoelectric detector. A carbon arc was used for absolute calibration of the monochromator-photomultiplier arrangement. The profiles have been scanned shot by shot, taking one shot about every 90 sec. A typical oscillogram is shown in Figure 2.

E. Results

The experimental values $\Delta\lambda_h$ of the halfwidths together with the calculated values $\Delta\lambda_{h1}$, $\Delta\lambda_{h2}$, and $\Delta\lambda_{h3}$ are given in Table 1. $\Delta\lambda_{h2}$ and $\Delta\lambda_{h3}$ obtained by straight line approximation, shows in many cases a surprisingly good agreement with the experimental values, in contrast to many observations on singly ionized atoms. The explanation is easy. The increased nuclear charge Z and main quantum number n of the visible and near UV lines cause the levels to be more hydrogen-like than those of neutral or singly ionized atoms [7]. Therefore the energy difference $\Delta E_{ii'}$ (being large compared to the line width $\hbar w$) is normally small compared to kT and the classical straight line model is well approximated. The walfwidth $\Delta\lambda_{h1}$ calculated with the semiempirical Gaunt-factor shows also a rough agreement, however the values are normally smaller than the observed ones.

Relatively strong deviations between calculated and observed values are obtained in the case of the $\lambda = 5696$ Å and especially the $\lambda = 4326$ Å line. Perhaps the normally neglected collisions causing transitions between the two C III term systems (ionization limits C IV ^{2}S and ^{2}P) are important for the broadening of these lines.

For the optically thin lines, the observed profiles fit a dispersion profile within the error limits in all cases. An example is given in Fig. 3 for the $3\,^2S_{1/2} - 3\,^2P_{1/2}$, $^2P_{3/2}$ doublet of C IV, which also shows very well the expected 2:1 intensity ratio and the equal halfwidth.

The interpretation of the optically thick C IV resonance line poses some difficulties, although the intensity of the line can be followed up over a wide wavelength range (Fig. 4). The exact dependence on $\Delta\lambda$ is difficult to determine on the far wings, because there are strong C III lines and other, non identified (perhaps also C III) lines. The line core, however, is not perturbed by other lines, and therefore the evaluation of $\Delta\lambda_h$ has been made in this wavelength region. The total halfwidth of the doublet was very reproducible at 5.5 Å ± 0.15 Å over

λ[Å]	Transition	$\Delta\lambda_h$	$\Delta\lambda_{h1}$	$\Delta\lambda_{h2}$	$\Delta\lambda_{h3}$
5696	$2s\,3p\,^1P - 2s\,3d\,^1D$	1.9	0.8	1.0	1.2
4647	$2s\,3s\,^3S_1 - 2s\,3p\,^3P_2$	0.95	0.55	0.8	0.9
4326	$2p\,3s\,^1P - 2p\,3p\,^1D$	2.1	0.6	0.9	1.1
4187	$2s\,4f\,^1F - 2s\,5g\,^1G$	4.1	3.8	3.6	3.7
3609	$2s\,4p\,^3P - 2s\,5d\,^3D$	6.2	7.6	7.5	7.1
2163	$2s\,3d\,^1D - 2s\,4f\,^1F$	0.46	0.39	0.47	0.45
1532	$2s\,3p\,^1P - 2s\,4d\,^1D$	0.43	0.44	0.49	0.44
5802	$3s\,^2S_{1/2} - 3p\,^2P_{3/2}$	1.6	0.84	1.35	1.45
5812	$3s\,^2S_{1/2} - 3p\,^2P_{1/2}$				
				$\Delta\lambda_{h4}$	
1548.2	$2s\,^2S_{1/2} - 2p\,^2P_{3/2}$	0.024	0.0064	0.02	
1550.8	$2s\,^2S_{1/2} - 2p\,^2P_{1/2}$				

Table 1. $\Delta\lambda_h$ = Experimental halfwidth in Å for an optically thin layer, $\Delta\lambda_{h1}$ = semiempirical halfwidth, $\Delta\lambda_{h2}$ = halfwidth from straight line approximation taking $v^{-1} = \overline{(1/v)}$, $\Delta\lambda_{h3}$ = halfwidth from straight line approximation taking $v = \overline{(v^2)}^{1/2}$, $\Delta\lambda_{h4}$ = halfwidth from hyperbolic orbit approximation. All values of $\Delta\lambda_h$ are given for $n_e = 4 \cdot 10^{17}$ cm^{-3}.

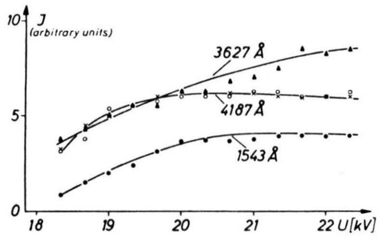

Fig. 1.

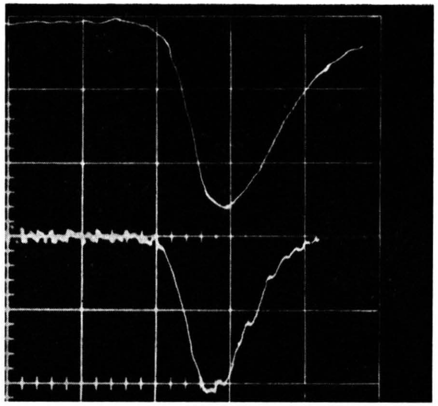

Fig. 2.

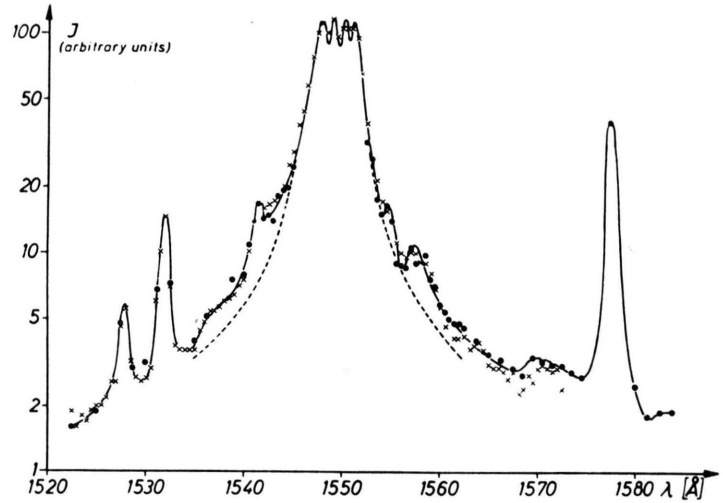

Fig. 4.

Fig. 1. Intensities at $\lambda=3627$ Å (continuum), $\lambda=4187$ Å (C III, two runs), and $\lambda=1543$ Å (C IV) as function of voltage of the condensers.

Fig. 2. Intensity at $\lambda=3210$ Å (He II) (upper curve) and $\lambda=1544$ Å (C IV) (lower curve) as function of time. Sweep 0.1 μs/div.

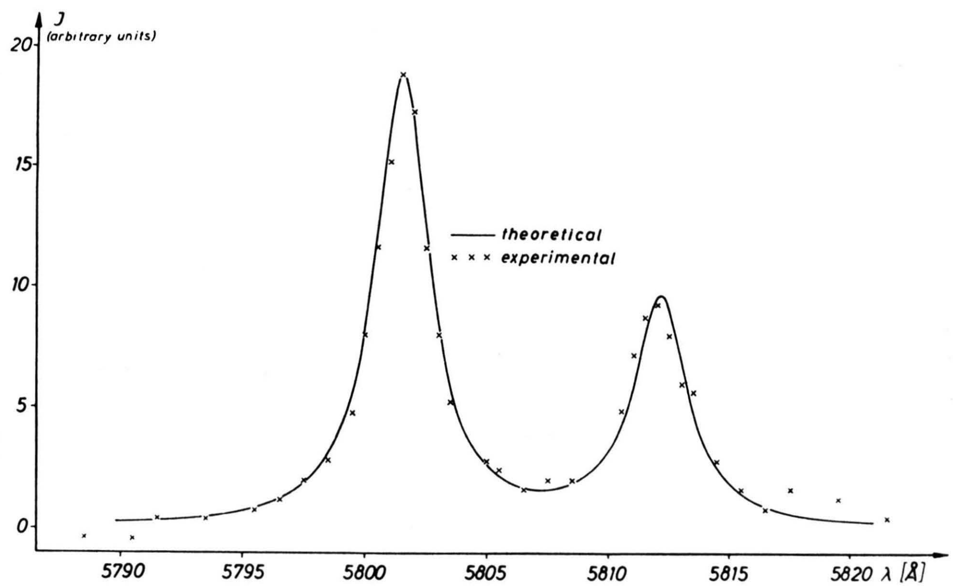

Fig. 3.

Fig. 3. The dispersion profile of the C IV $3\,^2S_{1/2}-3\,^2P_{1/2}$, $^2P_{3/2}$ doublet with $\Delta\lambda_h=2.7$ Å at $n_e=6.9\cdot10^{17}$ cm^{-3}. A constant background intensity of 6 units has been subtracted.

Fig. 4. Experimental profile of the C IV $2\,^2S_{1/2}-2\,^2P_{1/2}$, $^2P_{3/2}$ resonance doublet at $n_e = 4\cdot10^{17}$ cm^{-3}, $n_{i0}\cdot l = 1.6\cdot10^{17}$ cm^{-2}.
●—x—● Experimental profile with values of two different runs.
------ Dispersion profile for $\Delta\lambda_h=2.4\cdot10^{-2}$ Å and an estimated intensity background of about 2 units.

four runs. From this value we obtain $\Delta\lambda_h$ in the following way: First we find from the equation of radiative transfer for a homogeneous layer

$$I = B(1 - e^{-\tau})$$

the optical thickness at the halfwidth of an optically thick line

$$\tau = \log\left(\frac{B}{B-I}\right)\Big/\log e = \log 2/\log e = 0.69$$

(B = Kirchhoff-Planck function, $\tau = \varkappa \cdot l$ = optical thickness, $\varkappa$ = effective absorption coefficient = absorption minus induced emission coefficient, l = length of the optical layer = 30 cm).

Secondly we use the equation for τ of a collision broadened line [see [8], Eq. (68,16)] taking wavelength instead of angular frequency units and allowing induced emission

$$\tau = \frac{e^2\,\lambda^2}{2\,m\,c^2}\,n_{i0}\cdot f\,\frac{l\cdot\Delta\lambda_h}{\Delta\lambda^2 + (\Delta\lambda_h/2)^2}\left[1 - \exp\left(-\frac{h\,c}{\lambda\,k\,T}\right)\right].$$

Adding the optical thickness of the two lines

$$\lambda_1 = 1550.8\ \text{Å} \quad \text{and} \quad \lambda_2 = 1548.2\ \text{Å}$$

with a distance $\Delta\lambda_a = 2.573$ Å and neglecting $\Delta\lambda_h$ in the denominator we obtain

$$\tau = \frac{e^2\,\lambda^2}{2\,m\,c^2}\,n_{i0}\cdot l\left[\frac{f_1\,\Delta\lambda_h}{\Delta\lambda^2} + \frac{f_2\,\Delta\lambda_h}{(\Delta\lambda + \Delta\lambda_a)^2}\right]\left[1 - \exp\left(-\frac{h\,c}{\lambda\,k\,T}\right)\right].$$

In this equation we put the numerical values ($f_1 = 2\,f_2$, $f_2 = 0.095$ (see [4]), $\lambda = 1550$ Å, $l = 30$ cm, $n_{i0} = 5.2\cdot10^{15}\ \text{cm}^{-3}$.

With arbitrary $\Delta\lambda_h = x$, we draw the curve $\tau(\Delta\lambda)$. At the 5.5 Å width of the profile we measure τ^*. Then the experimental value of $\Delta\lambda_h = 2.4\cdot10^{-2}$ Å follows from $\tau^*/0.69 = x/\Delta\lambda_h$. This value is in satisfactory agreement with the calculated width [5] of $2\cdot10^{-2}$ Å. The accuracy of the theoretical value is not given in [5] for the case $\Delta E_{i'i} > 1$, the estimated error of the experimental value is in the order of 40%.

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